REXWINKLE

Investigation of

Starting Currents of Transformers

Electrical Engineering

B. S.

1912

INVESTIGATION OF STARTING CURRENTS OF TRANSFORMERS

BY

FRED DELONG REXWINKLE

THESIS

FOR THE

DEGREE OF BACHELOR OF SCIENCE

IN

ELECTRICAL ENGINEERING

COLLEGE OF ENGINEERING

UNIVERSITY OF ILLINOIS

1912

UNIVERSITY OF ILLINOIS

May 31, 1902

THIS IS TO CERTIFY THAT THE THESIS PREPARED UNDER MY SUPERVISION BY

FRED DELONG REXWINKLE

ENTITLED INVESTIGATION OF STARTING CURRENTS OF TRANSFORMERS

IS APPROVED BY ME AS FULFILLING THIS PART OF THE REQUIREMENTS FOR THE

DEGREE OF BACHELOR OF SCIENCE IN ELECTRICAL ENGINEERING

B. V. Jensen
Instructor in Charge

APPROVED: Ems Berg

HEAD OF DEPARTMENT OF ELECTRICAL ENGINEERING.

C O N T E N T S

INVESTIGATION OF STARTING CURRENTS OF TRANSFORMERS

Introduction

When the primary circuit of a transformer is first closed, there may be a transient current generated, due to the residual magnetism in the iron, and depending in magnitude upon the point on the e.m.f. wave at which the switch is closed. In the old style high frequency, low density transformer this effect was not very noticeable, but in modern transformers employing high densities the starting current may reach a value several times full load current with disastrous results.

The cause of this transient effect will be explained, and equations for calculating the starting current will be developed. The calculations for a given transformer will be made, and the proper method of protecting the system against this initial rush of current will be given.

Theory

If an alternating e.m.f. is impressed on a circuit containing resistance and reactance and not interlinked with any magnetic circuit, the impressed e.m.f. is consumed by the voltage drop due to resistance and the counter e.m.f. of self induction, and the flux generating the counter e.m.f. is proportional to the current flowing. Assuming the resistance drop to be negligible in comparison with the effect of the inductance, the current, and therefore the flux, lag 90 time degrees behind the impressed

e.m.f. and, starting with some point, such as zero on the e.m.f. wave, the flux has to change from its negative maximum to its positive maximum to produce the first half wave of counter e.m.f. If, then, the circuit is closed at the zero point of the e.m.f. wave, the flux will have to change by a value equal to 2 Φ_o, where Φ_o is the maximum value of flux. Since the flux starts at its zero point instead of its negative maximum, it must rise to a value equal to + 2 Φ_o, so that the current also will rise to twice its normal value. However, when this current flows, the effect of the resistance is no longer negligible, so that the current does not reach twice its normal value, but comes very near to it. In the next half wave the e.m.f. is reversed, but the current is still in the same direction, so that the counter e.m.f. must also cover the IR drop, and the flux must therefore change by more than 2 Φ_o. From this it is seen that the current drops to a value somewhat below the zero point, and in the third half wave the flux starts a little below zero, so that the current does not rise to quite such a high value. In the next half wave the current drops a little lower, and this process continues, until the current assumes its permanent value, the quicker the higher the resistance in proportion to the inductance. The curves show this action. In Figure 1 are shown the permanent e.m.f., current and flux waves, while Figure 2 shows the effect of starting the current at the zero point of the e.m.f. wave.

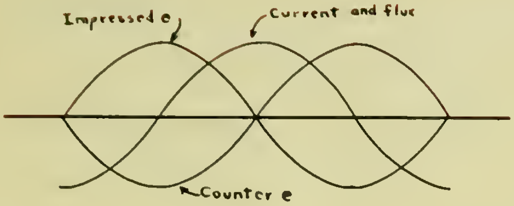


FIG 1.

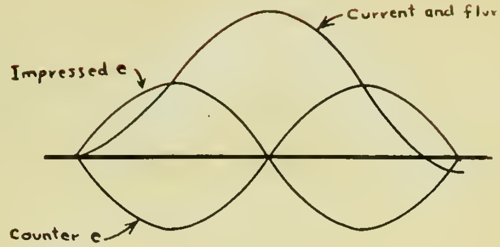


FIG. 2

By examining the curves it is seen that the maximum starting current is obtained, when the switch is closed at the zero degree point of the e.m.f. wave, and the minimum current is obtained, upon closing the switch, when the normal flux would be zero, that is, near the 90° point of the e.m.f. wave.

If the electric circuit is interlinked with a magnetic circuit, as in a transformer, there is usually a greater or less amount of residual magnetism in the iron, and the flux must therefore change by a value equal to $\Phi_r + 2\,\Phi$ where Φ_r is the residual magnetism. It is seen that, if the residual magnetism happens to be in the same direction in which the change of flux must take place, the flux will start, not at zero, but at a value equal to Φ_r, so that at the critical point, namely the zero point of the e.m.f. wave, the change of flux must be $+\Phi_r + 2\,\Phi_0$, and the current will reach a much higher value than without iron, especially in modern transformers in which the residual magnetism may be fairly high. Since the iron in these transformers is worked very near the knee of the saturation curve, it requires a large increase in magnetizing current to produce a comparatively small increase in flux as soon as the knee is passed. For a change of flux of $\Phi_r + 2\Phi_0$ therefore, the magnetizing current may reach formidable values.

Equations

The equations for calculating these starting currents are developed as follows: In the case of the circuit without iron let

r = the resistance,
L = the inductance,
x = the inductive reactance,
i = the current
e = E sin θ = the impressed e.m.f., when E is the maximum value of e.m.f. and θ is the phase angle of the impressed e.m.f.

Then, since the impressed e.m.f. is consumed by the ir drop and the counter e.m.f. of self induction,

$$e = E \sin\theta = ir + L\frac{di}{dt},$$

But

$$\frac{di}{dt} = \frac{di}{d\theta}\frac{d\theta}{dt} = \frac{di}{d\theta}2\pi f \text{ (since } \theta = 2\pi f t)$$

So

$$L\frac{di}{d\theta} = 2\pi f L\frac{di}{d\theta} = X\frac{di}{d\theta},$$

and

$$E\sin\theta = ir + x\frac{di}{d\theta}$$

$$E\sin\theta\, d\theta = ir\, d\theta + x\, di$$

$$di = \frac{E}{X}\sin\theta - \frac{ir}{X}d\theta$$

$$di = -\frac{E}{X}d(\cos\theta) - \frac{ir}{X}d\theta.$$

Assuming the ir drop to be negligible,

$$di = -\frac{E}{X}d(\cos\theta).$$

If the switch is closed at the 90° point, when the e.m.f. is at a maximum,

$$i = -\frac{E}{X}\int_{\pi/2}^{\pi} d(\cos\theta) = -\frac{E}{X}\left[\cos\theta\right]_{\pi/2}^{\pi} = +\frac{E}{X}$$ and is the maximum value of current obtained when the switch is closed at

this point. It is seen that this is the normal value of current, when the effect of resistance is not taken into account.

If, however, the switch is closed at the 0° point of the e.m.f. wave

$$i = -\frac{E}{X}\int_0^{\pi/2} d(\cos\theta) = -\frac{E}{X}\left[\cos\theta\right]_0^{\pi/2} = +\frac{E}{X}$$

which shows that in the first 90° the current just reaches its normal value, but, if the interval between 0 and π is taken, the current becomes

$$i = -\frac{E}{X}\int_0^{\pi} d(\cos\theta) = -\frac{E}{X}\left[\cos\theta\right]_0^{\pi} = +2\frac{E}{X}$$

which is just twice normal. Still, in this latter case the effect of resistance is two times normal, so that the current will not reach quite the value given by the equation. It is seen from these equations that the maximum value of the starting current in an air-cored coil will lie between normal and twice normal, depending upon the point of the e.m.f. wave at which the switch is closed and is not dangerous at any time.

In the iron clad circuit the flux is no longer proportional to the current, and when the iron becomes saturated, the increase in current causes an increase only in the flux through the air and non magnetic material between the coil and its core. As the flux density in the iron increases, the inductance decreases and remains at a small constant value due to the flux through the air. In this case the counter e.m.f. of self induction is expressed by $\frac{AdB}{d\theta}$, where B is the flux density in the iron and A is a constant.

Then

$$e = E\sin\theta = ir + A\frac{dB}{d\theta}$$

$$dB = -\frac{ir}{A}d\theta - \frac{E}{A}\,d(\cos\theta)$$

Under normal conditions the ir drop is negligible, so that

$dB = -\frac{E}{A}\,d(\cos\theta)$, and if the switch is closed at the 90° point, the normal magnetizing current will be obtained, and the maximum value of flux can be calculated by integrating dB from $\frac{\pi}{2}$ to π

$$\int_{\pi/2}^{\pi} dB = B_0 = -\frac{E}{A}\left[\cos\theta\right]_{\pi/2}^{\pi} = \frac{E}{A}$$

From the above,

$$B_0 = \frac{E}{A} \quad \text{or} \quad A = \frac{E}{B_0}$$

Then, in general,

$$dB = -\frac{B_0}{E}\,ir\,d\theta - B_0\,d(\cos\theta).$$

Method of Calculation

Since it is not possible to express the relation between the magnetizing current and the flux by any satisfactory equation, it is necessary to use analytical methods and obtain the magnetizing current from the saturation curve of the iron in the circuit. For the purpose of approximation increments will be substituted for the differentials,

so that

$$\Delta B = -B_0\,\Delta(\cos\theta) - \frac{B_0}{E}\,ir\Delta\theta,$$

and

$B = B' + \Delta B$, where B' is the flux density in the iron at the beginning of the interval $\Delta\theta$.

The method of calculation is as follows: Suppose the switch is closed at the zero point of the e.m.f. wave. Then, starting at zero, take increments of $\Delta\theta = 10^\circ = .175$ radians. Find B by taking the sum of the residual magnetism and the change of flux during the 10°, or $-B_0 \Delta (\cos \theta)$. The current corresponding can be read from the saturation curve, and the resistance effect calculated by $-B_0 \frac{ir \Delta\theta}{E}$ and subtracted from B. The new and more correct value of current can then be read from the curve. (This process is continued until the maximum current is obtained.) It is only at the higher values of current that the resistance has much effect on the values of B.

Calculations from Transformer Data

The calculations for a 7.5 KW transformer of modern design have been made to show the method of determining the starting current of any transformer and to give an idea of the magnitude of the transient effect in the latest transformers. The transformer tested is one of well known make and has special taps brought out for making the Scott connections for transforming from quarter phase to three phase. In this calculation it was used as a straight transformer.

For making the calculations the following data were obtained:

(a) Normal hysteresis loop,

(b) Saturation curve up to a straight line,

(c) Normal magnetizing current,

(d) Total resistance,

The hysteresis loop and magnetization curve were obtained as follows: Connections were made as shown in the diagram, Figure 3.

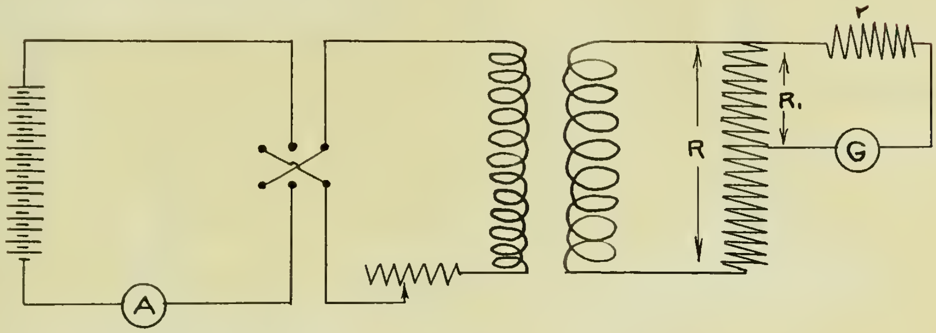


FIG. 3.

The current was supplied by a storage battery through a rheostat and a reversing switch to the primary and the secondary was connected to a high resistance (R). The galvanometer, one of the D'Arsonval type used ballistically, was shunted across a small part of the resistance (R) and also had a high resistance (r) connected in series with it to further cut down the current. The shunted part of the resistance (R_1) was adjusted so that the maximum value of current would not cause the galvanometer deflections to go off the scale. The values of (R), (R_1), and (r) were not changed during the experiment. A change of flux in the primary would cause a deflection of the galvanometer which was proprotional to the total change of flux. Since the deflections furnished a measure of the changed flux, it was unnecessary to find the constant of the circuit, as the actual values of the magnetizing current alone are desired.

The normal magnetizing current was determined by impressing normal voltage at normal frequency on the primary with the secondary on open circuit. The maximum value of this current was applied as direct current to obtain the normal hysteresis loop of the transformer. Firstly, the primary switch, Figure 3, was reversed several times with normal maximum magnetizing current (.44 amp.) flowing. Then a series of readings of current and galvanometer deflections were taken for the following cases: (a) as the switch was closed, (b) as the switch was opened, (c) as the switch was reversed, and (d) as it was opened again. This gave four points on the hysteresis loop as shown in Figure 4. The current was increased by convenient steps until the straight part of the saturation curve was reached. Several readings were taken at each value of current and the mean of them taken as the correct value of the deflections

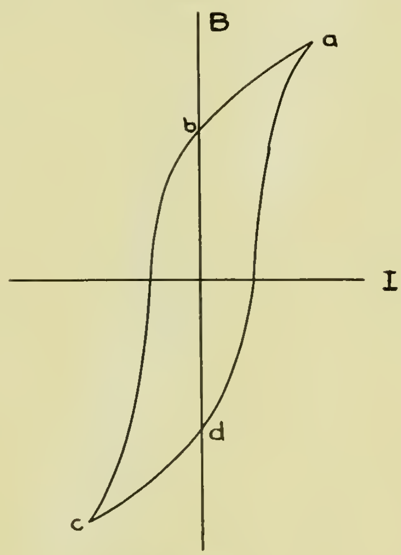


FIG. 4.

The method of working up the data for the saturation curve is shown below:

I	Change	Deflection	B_o	Residual Mag.
	a-b	3	$\frac{12.85+3}{2}$ =	7.93-3 =
	b-c	12.85		
.44	c-d	3	=7.93	=4.93
	d-a	12.85		

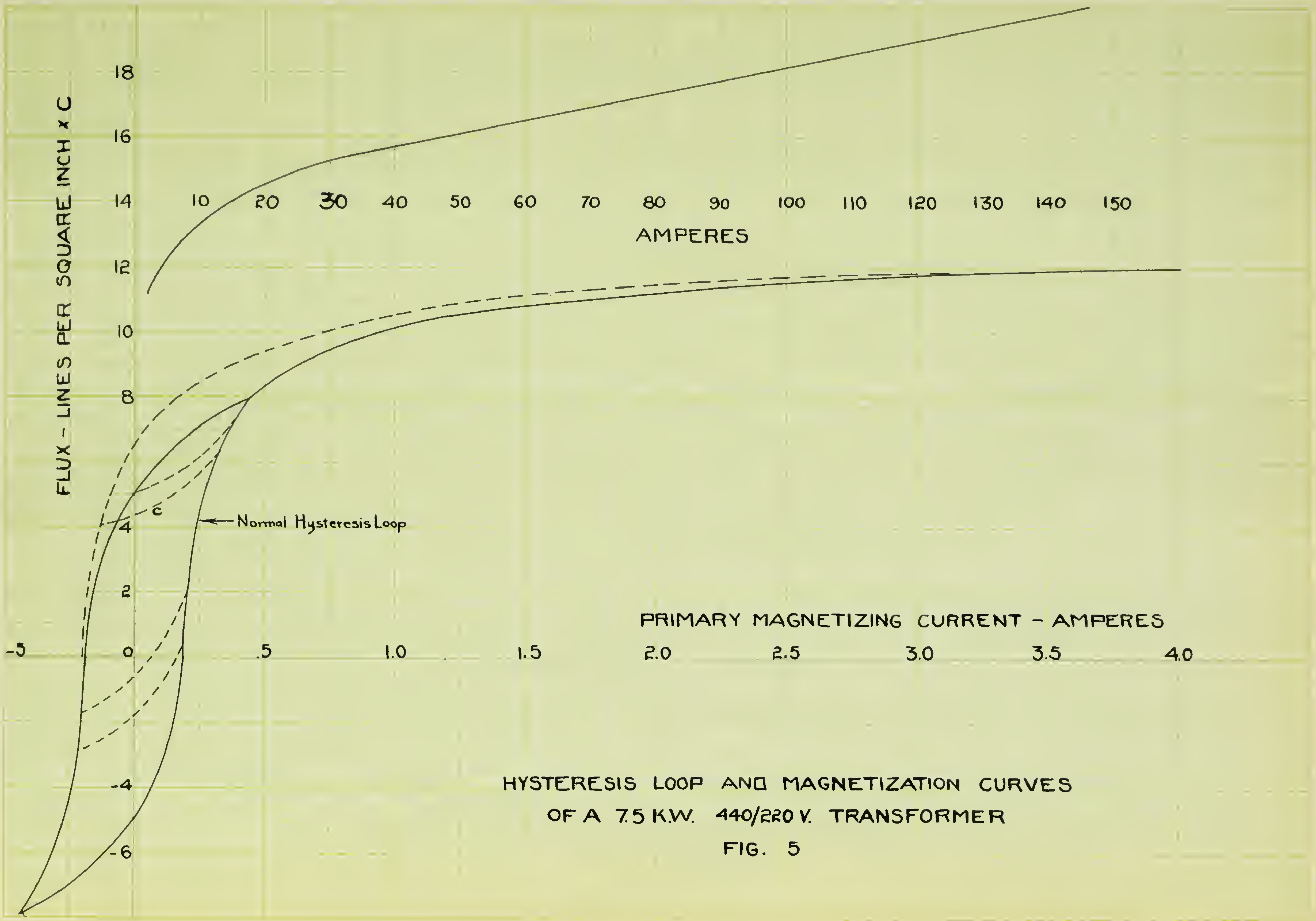
The calculations are for the normal value of magnetizing current and the other values were found in the same way.

TABLE I

SATURATION CURVE FOR 7 1/2 KW-440/220v. TRANSFORMER (440v. side)

NORMAL MAGNETIZING CURRENT .44 AMP. EFFECTIVE

I	Deflection		Φ_{max}	Residual Magnetism
.44	3	12.85	7.93	4.93
1.00	4.6	15.6	10.1	5.5
1.500	5.1	16.4	10.7	5.7
2	5.4	16.9	11.1	5.7
3.	5.8	17.55	11.67	5.87
4	6.1	18.00	12	5.9
5	6.4	18.3	12.3	5.9
12	7.6	19.7	13.6	6.0
15	7.9	20.1	14	6.1
20	8.4	20.7	14.5	6.1
25	8.7	21.3	15.0	6.2
30	9.0	21.4	15.2	6.2
35	9.3	21.5	15.4	6.1
40	9.4	21.8	15.6	6.2
50	9.7	22.7	16.2	6.5



HYSTERESIS LOOP AND MAGNETIZATION CURVES
OF A 7.5 K.W. 440/220 V. TRANSFORMER
FIG. 5

The current was run up to 50 amperes, when the curve assumed a straight line relation and was continued as far as necessary. The saturation curve is shown on the curve sheet, Figure 5. Table 1 gives the values for plotting the saturation curve as obtained in the manner explained above.

The calculations for the starting current were made only for the extreme cases. That is, at the 90° point, where the current is a minimum, and at the 0° point, where it is a maximum, the residual magnetism in each case being + B_r. Since the resistance drop at the higher currents becomes appreciable, it was necessary to make two or three trials at each of the higher values of current before the equation $[\Delta B = -B_o \Delta(\cos\theta) - \frac{B_o}{E} ir \Delta\theta]$ was satisfied. In starting at the value of residual magnetism equal to +4.93, it was necessary to go up along the dotted curve until the regular curve was reached, and for the descending values of the flux it was necessary to run down on the upper dotted curve on account of the tendency of the magnetism to remain in the iron. It is understood that all the values of flux as used here are to be multiplied by a constant to obtain the real flux.

The constants obtained for the transformer were:

Primary volts, 440 volts, effective,

Normal magnetizing current, .44 amperes, maximum

Primary resistance, .2 ohm.

The equation now takes the following form:

$$\Delta B = -B_o \Delta(\cos\theta) - \frac{B_o}{E} ir \Delta\theta$$

TABLE 2

CALCULATION OF STARTING CURRENT - RESISTANCE OF LEADS NEGLIGIBLE (RESIDUAL MAGNETISM +4.93) K = .000445

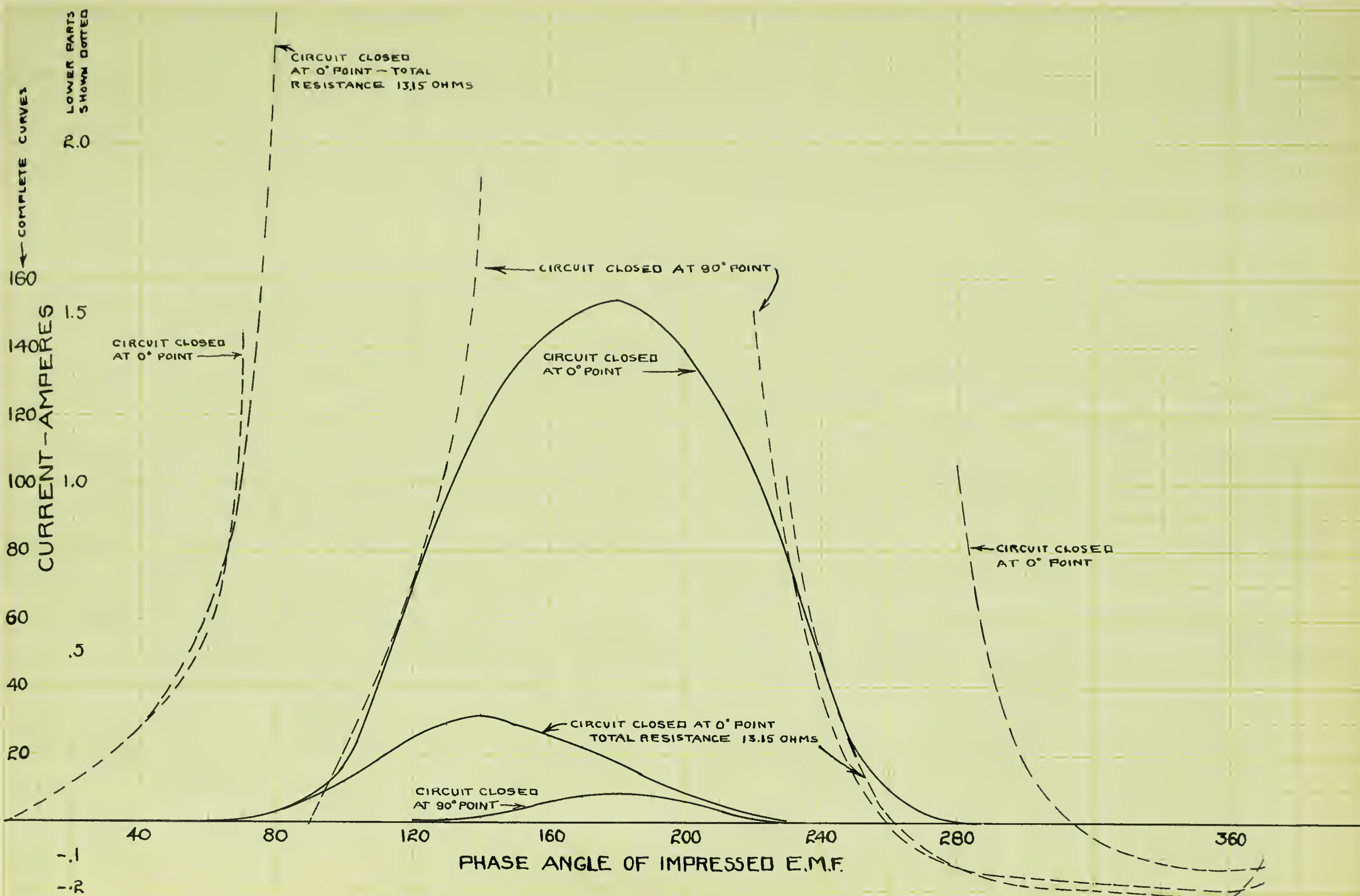
	Circuit Closed at 0^{o} point						Circuit Closed at 90^{o} Point		
θ	Cos θ	ΔB_1	ΔB	B	i	ki	ΔB	B	i
0	+1.00			4.93	0				
10	.98	+.16	+.16	5.09	.05				
20	.94	.32	.32	5.41	.12				
30	.87	.55	.55	5.96	.15				
40	.77	.79	.79	6.75	.30				
50	.64	1.03	1.03	7.78	.42				
60	.50	1.11	1.11	8.89	.51				
70	.34	1.27	1.27	10.16	1.45				
80	+.17	1.35	1.35	11.51	2.7				
90	0	1.35	1.35	12.86	8			+4.93	0
100	-.17	1.35	1.34	14.20	17	.01	+1.35	6.28	+.25
110	.34	1.35	1.33	15.53	40	.02	1.35	7.63	.42
120	.50	1.27	1.24	16.77	68	.03	1.27	8.90	.675
130	.64	1.11	1.07	17.84	94	.04	1.11	10.01	1.05
140	.77	1.03	.98	18.82	118	.05	1.03	11.04	1.90
150	.87	.79	.73	19.55	134	.06	.79	11.83	3.65
160	.94	.55	.49	19.94	144	.06	.55	12.38	6.00
170	.98	.32	.25	20.19	150	.07	.32	12.70	7.50
180	1.00	+.16	+.09	20.28	154	.07	+.16	12.86	8.50
190	.98	-.16	-.23	20.05	147	.07	-.16	12.70	7.50
200	.94	.32	.38	19.67	139	.06	.32	12.38	6.00
210	.87	.55	.61	19.06	124	.06	.55	11.83	3.65
220	.77	.79	.84	18.22	104	.05	.79	11.04	1.50
230	.64	1.03	1.06	17.16	77	.03	1.03	10.01	.775
240	.50	1.11	1.13	16.03	51	.02	1.11	8.90	.375
250	.34	1.27	1.28	14.75	23	.01	1.27	7.63	+.14
260	-.17	1.35	1.35	13.30	10.05		1.35	6.28	-.01
270	0	1.35	1.35	11.95	3.9		1.35	4.93	.09
280	+.17	1.35	1.35	10.60	1.05		1.35	3.58	.14
290	.34	1.35	1.35	9.25	.475		1.35	2.23	.15
300	.50	1.27	1.27	7.98	.175		1.27	-.96	.16
310	.64	1.11	1.11	6.87	.04		1.11	-.15	.165
320	.77	1.03	1.03	5.84	-.04		1.03	1.18	.175
330	.87	.79	.79	5.05	-.08		.79	1.97	.18
340	.94	.55	.55	4.50	-.11		.55	2.52	.185
350	.98	.32	.32	4.18	-.125		.32	2.84	.19
360	1.00	- .16	-.16	4.02	-.125		-.16	3.00	.2
370	.98	+ .16	+.16	4.18	-.125		+.16	-2.84	-.175

TABLE 3

CALCULATION OF STARING CURRENT - TOTAL RESISTANCE OF PRIMARY AND LEADS 13.15 OHMS - RESIDUAL MAGNETISM +4.95

K = .0293

θ	$\cos\theta$	ΔB_1	ΔB	B	i	Ki
0	+1.00			+4.93	0	
10	.98	+.16	+.16	5.09	+.05	
20	.94	.32	.32	5.41	.12	
30	.87	.55	.55	5.96	.15	
40	.77	.79	.79	6.75	.30	
50	.64	1.03	1.02	7.77	.42	.01
60	.50	1.11	1.09	8.86	.62	.02
70	.34	1.27	1.24	10.10	1.00	.03
80	+.17	1.35	1.28	11.38	2.4	.07
90	0	1.35	1.16	12.54	6.4	.19
100	-.17	1.35	1.00	13.54	12.0	.35
110	.34	1.35	.80	14.34	18.5	.55
120	.50	1.27	.54	14.88	15.0	.73
130	.64	1.11	.26	15.14	29.0	.85
140	.77	1.03	+.09	15.24	32.0	.94
150	.87	.79	-.06	15.18	29.0	.85
160	.94	.55	.20	14.98	26.0	.75
170	.98	.32	.34	14.64	22.5	.66
180	1.00	+.16	.37	14.27	18.0	.53
190	.98	-.16	.54	13.73	13.0	.38
200	.94	.32	.60	13.13	9.5	.28
210	.87	.55	.74	12.39	6.5	.19
220	.77	.79	.86	11.53	2.5	.07
230	.64	1.03	1.06	10.47	1.0	.03
240	.50	1.11	1.12	9.35	.5	.01
350	.34	1.27	1.28	8.08	.2	
260	-.17	1.35	1.35	6.73	+.025	
270	0	1.35	1.35	5.38	-.065	
280	+.17	1.35	1.35	4.03	.125	
290	.34	1.35	1.35	2.68	.165	
300	.50	1.27	1.27	1.41	.18	
310	.64	1.11	1.11	+ .30	.19	
320	.77	1.03	1.03	- .73	.19	
330	.87	.79	.79	1.52	.195	
340	.94	.55	.55	2.07	.20	
350	.98	.32	.32	2.39	.21	
360	1.00	-.16	-.16	2.55	.215	
370	+.98	+.16	+.16	-2.39	-.09	



STARTING CURRENTS OF A 7.5 K.W. TRANSFORMER

FIG. 6

$$\Delta B = -7.93\,\Delta(\cos\theta) - \frac{7.93 \times .2 \times .175}{440\sqrt{2}} \times i$$

$$\text{or,}\quad \Delta B = -7.93\,\Delta(\cos\theta) - .000445\,i, \text{ when } \theta = 10^\circ$$

The calculations for the starting currents, when the switch is closed on the primary side with the secondary open, are shown in Table 2, and the corresponding curves are shown on the curve sheet, Figure 6. It is seen that the current rises to a value about six and one-half times full load when the circuit is closed at the 0° point, while it is only about one-third full load when the circuit is closed at the 90° point. It was found that the resistance had no appreciable affect when the current was below 12 amperes. It will be noticed that at the 370° point the flux starts back along the dotted line as indicated by c, Figure 5, and the current, therefore, approaches its normal value very slowly.

It can readily be understood how dangerous this transient effect would be in a station employing large step-up transformers and several sets of generators connected to one constant potential bus bar. In a station of this kind it would be impossible to bring up the voltage slowly on each transformer as it becomes necessary to connect it in, and if the switch happened to be closed at or near the 0° point of the e.m.f. wave the effect would be almost equivalent to a short circuit on the machine, and in the case of generators of low reactance, such as turbo-alternators, serious damage might result.

Protection Against Starting Currents

It was seen in making the first calculations that the resistance drop cut down the starting current considerably at the higher values, so calculations have also been made to show the effect of inserting resistance in the circuit. The resistance was so chosen that the total resistance of the primary circuit up to the source of constant potential was sufficient to cause a drop of 1/2 normal voltage at full load current, so that the counter e.m.f. of the transformer had to take care of only a part of the impressed e.m.f. The calculations for this case are shown in Table 3 and the corresponding curve is shown in Figure 6 to allow comparison with the other results. The maximum value of current when the switch is closed at the 0° point is only 1 1/3 times full load and the transformer can easily stand this for a long time. Since the current will assume reasonable values in a fraction of a second, or a few cycles, the resistance need be in series only a short time. This can be accomplished as shown in Figure 7, which shows an extra contact on the switch so arranged that it will make contact a little before the regular contact is made.

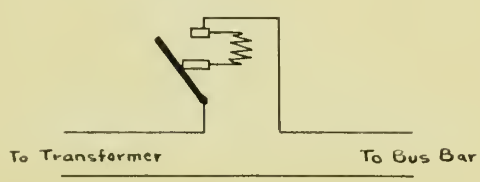


FIG. 7

The resistance is automatically cut out as the switch is closed, and ample protection is afforded by this simple device.

Conclusions

From the foregoing it is seen that a heavy transient current may be generated in modern transformers on first closing the primary circuit. This effect is much more pronounced in transformers of low frequency, as they usually operate at higher density and nearer the knee of the saturation curve than do those of high frequency. It is also shown that this starting current can be cut down to reasonable values by making the resistance in the leads at least enough to furnish a drop equal to one-half normal voltage until the transient effect has been reduced sufficiently to permit the external resistance to be removed.